

All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated.

A scientific calculator is allowed.

Student Name: KEY

ID: _____

Instructor: _____

Exam Score: _____

1. Find the first derivative of the following functions.

a. $f(x) = \arctan x^3$

$$f'(x) = \frac{3x^2}{1 + (x^3)^2} = \frac{3x^2}{1 + x^6}$$

b. $g(x) = \int_0^{\sin x} \sqrt{t^2 + 1} dt$

$$g'(x) = \cos x \sqrt{\sin^2 x + 1}$$

c. $f(x) = 7^{x^3} \cdot \sinh x$

$$f'(x) = 3x^2(\ln 7) 7^{x^3} \sinh x + 7^{x^3} \cosh x$$

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2. Find the indefinite integrals.

a. $\int \frac{6}{9+25x^2} dx$ $u^2 = 25x^2$, $u = 5x$, $a = 3$
 $du = 5dx$
 $\frac{1}{5} du = dx$

$$\frac{1}{5} \int \frac{6}{3^2 + u^2} du = \frac{6}{5} \left(\frac{1}{3} \right) \arctan \frac{u}{3} + C$$
$$= \frac{2}{5} \arctan \left(\frac{5x}{3} \right) + C$$

b. $\int \frac{x^3 - 2x^2 + 3}{x-2} dx$

$$\begin{array}{r} 2 \overline{) 1 \quad -2 \quad 0 \quad 3} \\ \underline{ 2 \quad 0 \quad 0} \\ 1 \quad 0 \quad 0 \quad 3 \end{array}$$

$$\int x^2 + \frac{3}{x-2} dx = \frac{1}{3} x^3 + 3 \ln|x-2| + C$$

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3. Find the derivative of the following function using logarithmic differentiation.

$$y = \frac{x^2 \sqrt{3x+2}}{(x^2-1)^2}, \quad x > -\frac{2}{3}$$

$$\ln y = \ln \left[\frac{x^2 (3x+2)^{1/2}}{(x^2-1)^2} \right]$$

$$\ln y = 2 \ln x + \frac{1}{2} \ln(3x+2) - 2 \ln(x^2-1)$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{1}{2} \left(\frac{3}{3x+2} \right) - 2 \left(\frac{2x}{x^2-1} \right)$$

$$y' = \frac{x^2 \sqrt{3x+2}}{(x^2-1)^2} \left[\frac{2}{x} + \frac{3}{6x+4} - \frac{4x}{x^2-1} \right]$$

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4. Find the area under the curve $f(x) = e^{\cot x}(\csc x)^2$ between $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$.

$$A = \int_{\pi/4}^{\pi/2} e^{\cot x} (\csc^2 x) dx$$

$$u = \cot x$$

$$du = -\csc^2 x dx$$

$$-du = \csc^2 x dx$$

$$-\int_1^0 e^u du = \int_0^1 e^u du = e^u \Big|_0^1$$

$$= e - e^0 = e - 1$$